Minimizing the effect of sinusoidal trends in detrended fluctuation analysis

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Abstract

The detrended fluctuation analysis (DFA) [Peng et al., 1994] and its extensions (MF-DFA) [Kantelhardt et al., 2002] have been used extensively to determine possible long-range correlations in self-affine signals. While the DFA has been claimed to be a superior technique, recent reports have indicated its susceptibility to trends in the data. In this report, a smoothing filter is proposed to minimize the effect of sinusoidal trends and distortion in the log-log plots obtained by DFA and MF-DFA techniques.

1   Introduction

There have been several instances where data recorded from a wide variety of systems exhibit broadband power law decay [Bassingthwaite et al, 1995]. These include the class of self-similar (self-affine) data sets that lack a well-defined temporal scale. The nature of the decay is reflected by the scaling exponent . Such self-similar data sets can be broadly classified into monofractal and multifractals [Stanely et al., 1999]. While the former is characterized by a single scaling exponent (α), the latter has a spectrum of scaling exponents. Identifying the scaling exponents has been found to be of practical value in distinguishing the various states of activities [Ivanov et al., 1999]. Thus developing suitable techniques to accurately capture the scaling exponents has been an area of active research. Peng et al. (1994) introduced the detrended fluctuation analysis (DFA, Appendix (A)) and demonstrated its superiority over other traditional techniques such as re-scaled range and Hurst analysis in estimating the scaling exponent. A scaling exponent ($0 < \alpha < 0.5$) is characteristic of anti-persistent behavior, whereas that of ($0.5 < \alpha < 1$) indicates persistent behavior or long-range correlations in the data. Kantelhardt et al. (2002) extended the DFA technique (MF-DFA, Appendix (A)) to capture the spectrum of exponents in the case of multifractal data. The ease of implementation and interpretation of results obtained from DFA and MF-DFA has enjoyed popularity among a wide spectrum of researchers from diverse disciplines. However, recent studies [Kanterlhardt et al., 2001; Hu et al., 2001] have pointed out the susceptibility of the (DFA) to trends in the data.

Kantelhardt et al, (2001) suggested that DFA-k which incorporates the k 'th order polynomial detrending has been found to be immune to polynomial trends lesser than k. While the procedure lends

itself to be robust to polynomial trends this not true for other types of trends. Periodic trends are quite common in experimental data sets, such as diurnal cycles, temperature fluctuations and seasonal effects. The traditional DFA [Peng et al., 1994], assumes that the scaling of the fluctuation $F(s)$ versus the time scale (s) is in the form of a power law fashion with a single exponent, i.e. $F(s) \sim s^{\alpha}$, where the scaling exponent (α) is estimated by a linear regression of the log-log plot. There have been instances where the log-log plot exhibit more than one scaling region. Such a phenomenon has been termed as a crossover [Peng et al., 1995] and may be used as a pre-cursor to identifying possible multifractal structure. Hu et al, (2001) pointed out spurious crossovers in the log-log fluctuation plots which can arise due to of sinusoidal trends. Thus it is important to develop techniques to understand whether the crossover is due to the existence of multiple scaling exponents in the dynamics or a manifestation of the extrinsic sinusoidal trend. As we shall demonstrate, polynomial detrending of finite order might not be useful in removing the sinusoidal trends. This is to be expected as the Maclaurin's series expansion of a sinusoidal function results in infinite terms. The present study addresses the above problem and proposes a combination of smoothing filter in the frequency domain and q'th generalized moment estimation to overcome the above problem. While the former minimizes the effect of the crossovers introduced by the sinusoidal trend, the latter provides a qualitative understanding of the nature of the crossover.

2 Minimizing the effect of sinusoidal trends

Sinusoidal trends superimposed on a broad-band power-law noise can be identified by characteristic peaks in the power spectrum. A simple averaging technique in the frequency domain can be useful in eliminating the effect of trends while retaining the power-law behavior of the noise. A smoothing filter to accomplish this is explained below.

Smoothing Filter :

1. Let the power-law noise be represented by x and the sinusoidal trend by t. Therefore the noise with the trend is given by $y = x + t$; with Fourier transform $F(f)$.

2. Let the frequency in the Fourier transform corresponding to the periodic trend occur at $f = f_k$.

3. Replace the power at the frequency f_k by a smoothing filter

$$|F^*(f_k)| = 0.5(|F(f_{k-1})| + |F(f_{k+1})|)$$
$$|F^*(f)| = |F(f)| \ , f \neq f_k$$

Assign random phases so as to satisfy the conjugacy constraints.

4. Determine the inverse Fourier transform to obtain the filtered data y^f.

If the objective is to minimize the power contributed by the trend, randomizing the phases (Step 4) might not be necessary as the power spectrum and hence, the auto-correlation is immune to the phase information in the signal (Weiner-Khintchine theorem) [Proakis and Manolakis, 1992]. In the following case studies, we shall utilize the smoothing filter to minimize the effect of sinusoidal trends on the DFA and MF-DFA estimation procedures. We consider synthetic and real world data sets.

Case (i) Monofractal long- range correlated noise corrupted with sinusoidal trends

In [Hu et al., 2001], the susceptibility of the DFA (1) to the monofractal data infected with sinusoidal trend was discussed. It was concluded that the plot of $log(F(s))$ vs $log(s)$ exhibited a crossover proportional to the time period of the sine wave. To determine the effectiveness of the smoothing filter we use the data published in [Hu et al., 2001]. The power spectrum of a power-law noise superimposed by a sinusoidal trend with amplitude $A = 2$ and period $T = 2^7$ is shown in Fig 1a, Appendix (B). The DFA of the power-law noise for the various order polynomial detrending is shown in Fig 1b. As expected the scaling exponent from the log-log plot is $\alpha = 0.9$, conforming to earlier reports [Hu et al., 2001]. However, the log-log plot of the power-law noise with sinusoidal trend exhibits a characteristic crossover, Fig 1c. It is important to note that the crossover is due to the trend and not the dynamics. Despite higher order polynomial detrending ($d = 1, 2, 3, 4, 5$, and 6), the spurious crossover induced by the sinusoidal persists, see Fig 1c. The distortion introduced by the sinusoidal trend imparts a nonlinear structure to the fluctuation function and prevents the application of linear regression to estimate the scaling exponent. However, the fluctuation plots obtained after applying the smoothing filter resembles straight lines with ($\alpha \sim 0.9$), Fig 1d, similar to that of the trend free power law noise, Fig 1b. Thus the spurious nonlinear structure introduced by the sinusoidal trend is minimized on applying the smoothing filter prior to the DFA estimation procedure.
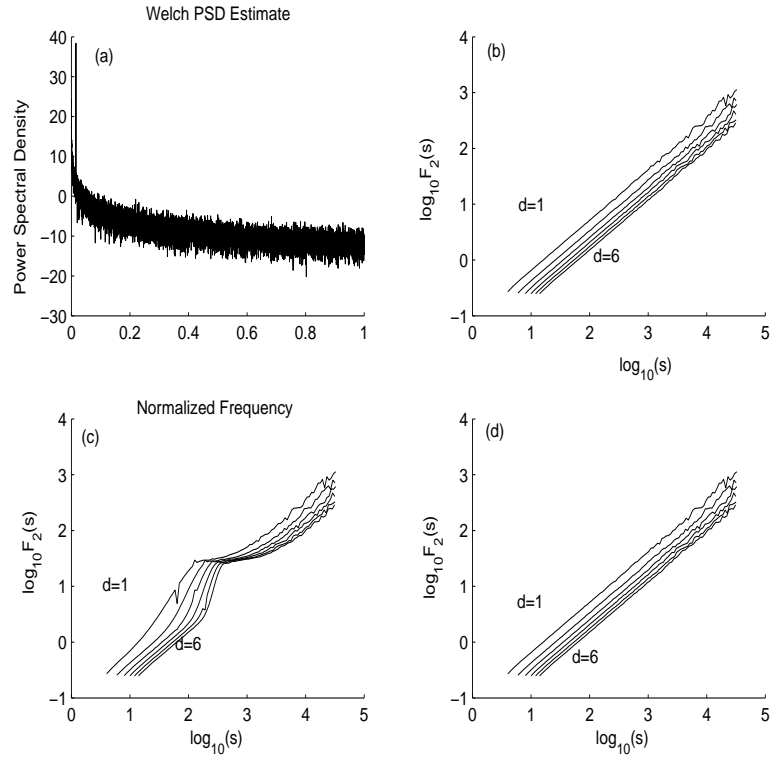

Figure 1: Power spectrum of monofractal data ($\alpha = 0.9$) corrupted with sinusoidal trend is shown in (a). The log-log plot of the fluctuations obtained by applying the second-order DFA ($q = 2$) for the various polynomial detrendings ($d = 1, 2, 3, 4, 5$ and 6) for the monofractal data, monofractal corrupted with sinusoidal trend, and that reconstructed using the smoothing filter is shown in (b, c and d) respectively.

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Case (ii) Monofractal long-range correlated noise corrupted with multiple sinusoidal trends

To determine the effect of multiple sinusoidal trends, the power-law noise ($\alpha = 0.9$) was corrupted with sinusoidal trends with parameters ($A_1 = 6, A_2 = 3, A_3 = 2, T_1 = 2^6, T_2 = 2^4; T_3 = 2^2$, Appendix (B). The frequencies ($f_i = 1/T_i$) of the trends are chosen such that f_2 and f_3 represent the second and the third harmonic of the fundamental f_1. Harmonic trends are commonly observed in experimental data hence its discussion in the present study. The power spectrum of the power law noise with multiple sinusoidal trends is shown in Fig 2a. The corresponding log-log plot of the fluctuation is shown in Fig 2b and resembles that obtained in the case (i). Similar to case (i), the smoothing filter is useful in minimizing the effect of the trends introduced by the sinusoidal trends and thereby facilitates a reliable extraction of the scaling exponent ($\alpha \sim 0.9$). The traditional DFA captures only the second moment ($q = 2$, i.e. $F_2(s)$) similar to power spectral techniques. To examine the effect of the generalized moments (q), the fluctuations were computed with polynomial order ($d = 4$) and varying $q = (-10, -8, -6, -4, -2, 2, 4, 6, 8, 10)$. The log-log plot of the fluctuations $F_q(s)$ with respect to the time scale s with varying q on the power law noise corrupted with harmonic trends is shown in Fig 2d. While the log-log plot fails to exhibit a linear trend, the nature of the fluctuations does not change appreciably with varying q. Thus, in the absence of the smoothing filter, determining the qualitative behavior of the fluctuation for various values of q can provide insight into the nature of the crossover in the given data. Such an analysis can be helpful in the case of more complex dynamics such as multifractal noise as discussed below.
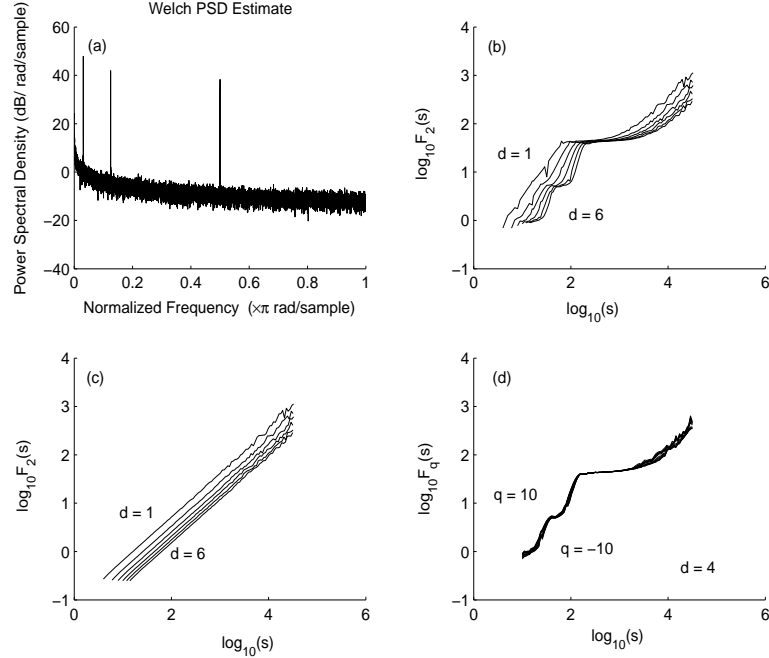

Figure 2: Power spectrum of monofractal data ($\alpha = 0.9$) corrupted with sinusoidal trends is shown in (a). The log-log plot of the fluctuations obtained by applying the second-order DFA ($q = 2$) for the various polynomial detrendings ($d = 1, 2, 3, 4, 5$ and 6) for the monofractal corrupted with sinusoidal trend and that reconstructed by the smoothing filter is shown in (b and c) respectively. The fluctuation $F_q(s)$ obtained for ($q = -10, -8, -6, -4, -2, 2, 4, 6, 8, 10$) with ($d = 4$) is shown in (d).

Case (iii) Multifractal noise corrupted with multiple sinusoidal trends

The multifractal data considered is that of internet traffic [Levy Vehel and Reidi, 1996]. The data was corrupted with sinusoidal trends with amplitude and the time period $(A_1 = 6000, A_2 = 3000, T_1 = 2^6, T_2 = 2^4$, Appendix (C). As in case (ii), the frequencies are harmonically related to one another. The MF-DFA of the multifractal data estimated with polynomial order $(d = 4)$ and varying $q = (-10, -8, -6, -4, -2, 2, 4, 6, 8, 10)$ is shown in Fig. 3b. The log-log plots exhibit a marked change in the slope characteristic of multifractal data. The log-log plot of the multifractal data after applying the smoothing filter is shown in Fig. 3d and resembles that of Fig 3b. Fig. 3c shows the log-log fluctuation of the multifractal data with the sinusoidal trend. It can be observed unlike case (ii), Fig. 2d, the slopes show a dramatic change with varying q. Thus even in the absence of a smoothing filter the analysis of the log-log with varying q can provide insight into the nature of the dynamics. However, the estimation of the slopes for multifractal data is a non-trivial issue and not discussed in the present study.
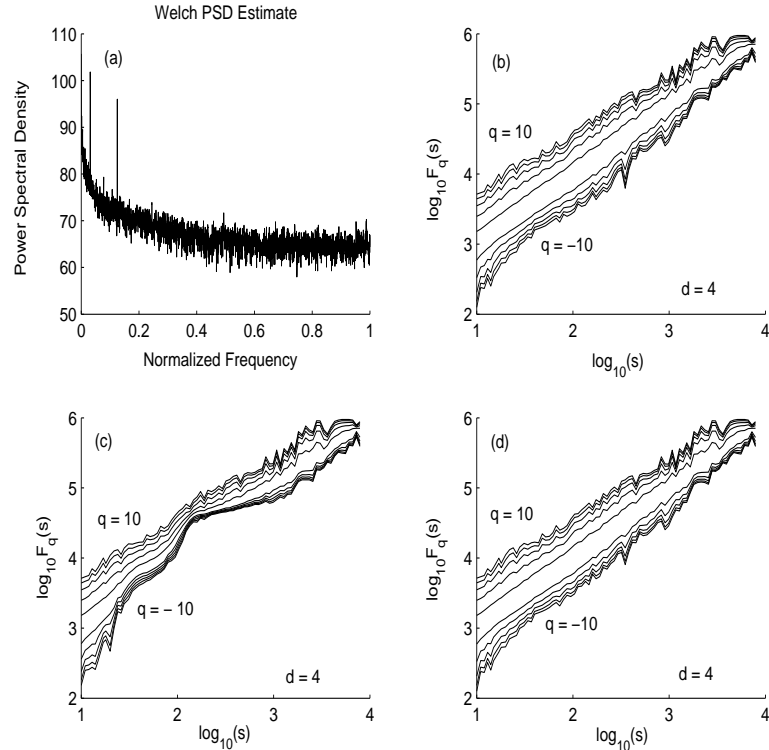

Figure 3: Power spectrum of multifractal data corrupted with sinusoidal trends is shown in (a). The fluctuation $F_q(s)$ obtained for $(q = -10, -8, -6, -4, -2, 2, 4, 6, 8, 10)$ with $(d = 4)$ for the multifractal data, multifractal data corrupted with sinusoidal trends, and that obtained by a smoothing filter is shown in (b, c, and d) respectively.

3 Conclusions

The retention of sinusoidal trends despite higher order polynomial detrending can induce spurious crossovers and prevent reliable extraction of scaling exponents in DFA/MF-DFA procedures. In the present report, a simple smoothing filter is proposed to minimize the effect of sinusoidal trends on the

DFA/MF-DFA procedures. The effectiveness of the smoothing filter on monofractal and multifractal data corrupted with sinusoidal trends were considered. While the smoothing filter minimizes the trends it is nevertheless an approximation. A qualitative approach to determine whether the observed distortion in the log-log plots is an outcome of the intrinsic dynamics or trend is also discussed. It is important to note that only sinusoidal trends whose power is much smaller than that of the power-law noise were discussed in the present study.

References

Bassingthwaighte., J.B. Liebovitch., L.S. West., B.J. [1995] *Fractal Physiology*, Oxford Univ. Press.

Hu., K. Ivanov., P.C.h. Chen., Z. Carpena., P. and Stanley H.E. [2001] "Effect of trends on detrended fluctuation analysis," *Phys. Rev. E.* **64**, 011114 (19 pages).

Ivanov., P. Ch. Amaral., L. A. N. Goldberger., A. L. Havlin., S. Rosenblum., M. G. Struzik., Z. R. and Stanley., H. E. [1999] "Multifractality in human heartbeat dynamics," *Nature* **399**, 461-465.

Kantelhardt., J.W. Koscielny-Bunde., E. Rego., H.A.H. Havlin., S. Bunde., A. [2001] "Detecting Long-range Correlations with Detrended Fluctuation Analysis," *Physica A*, **295**, 441-454.

Kantelhardt., J.W. Zschiegner., S.A. Koscielny-Bunde., E. Havlin., S. Bunde., A. and Stanley., H.E. [2002] "Multifractal detrended fluctuation analysis of nonstationary time series," *Physica A* **316**, 87.

Levy Vehel., J. and Reidi., R. H. [1996] Fractional Brownian motion and data traffic modeling: The other end of the spectrum, *Fractals in Engineering*, (Eds. Levy Vehel, J. Lutton, E. and Tricot, C.) Springer-Verlag.

Peng., C-K. Buldyrev., S.V. Havlin., S. Simons., M. Stanley., H.E. Goldberger., A.L. [1994] "On the mosaic organization of DNA sequences," *Phys. Rev. E.* **49**, 1685-1689.

Peng., C-K. Havlin., S. Stanley., H.E. Goldberger., A.L. [1995] "Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series," *Chaos* **5**, 82-87.

Proakis., G. and Manolakis., D. [1992] "Digital Signal Processing, Principles, Algorithms, and Applications", 2nd Ed. Macmillan.

Stanely., H.E. Amaral., L.A.N. Goldberger., A.L. Havlin., S. Ivanov., P.Ch. Peng., C-K. [1999] "Statistical physics and physiology: Monofractal and multifractal approaches", *Physica A* **270**, 309-324.

Appendix A : Multifractal Detrended Fluctuation Analysis (MFDFA)

A brief description of the Multifractal DFA (MF-DFA) [Kantelhardt et al., 2002] is included below for completeness of the report. The scaling exponent in the case of monofractal data can be obtained by fixing $(q = 2)$.

1. The integrated series of the given data $\{x_k\}$ is generated as

$$y(k) = \sum_{i=1}^{i=k} [x(i) - \bar{x}] \quad k = 1, \ldots N$$

   where $\bar{x}$ represents the average of $\{x_k\}, k = 1 \ldots N$

2. The data is divided in to n_s non-overlapping boxes of equal lengths where $n_s = int(N/s)$. The local polynomial trend $y_v(i)$ is calculated in each of the bins $v = 1 \ldots n_s$ by a polynomial regression and the variance is determined from

$$F^2(v, s) = \{\frac{1}{s} \sum_{i=1}^{i=s} \{y[N - (v - n_s)s + i] - y_v(i)\}^2$$

   The order m of the polynomial trend is chosen such that $s \geq m + 2$. Polynomial detrending of order m is capable of eliminating trends up to order $m - 1$. This procedure is repeated from the other end of the data in order to accommodate all the samples. Thus the effective length of the data is $2n_s$.

3. The qth order fluctuation function is calculated from averaging over all segments.

$$F_q(s) = \left\{ \frac{1}{2n_s} \sum_{i=1}^{i=2n_s} [F^2(v, s)]^{q/2} \right\}^{1/q}$$

   In general, the index q can take any real value except zero, [Kantelhardt et.al., 2002]. The scaling behavior is determined by analyzing the log-log plots $F_q(s)$ versus s for *each* q. If the original series $\{x_k\}$ is power-law correlated, the fluctuation function will vary as

$$F_q(s) \sim s^{h(q)}$$

A change in the shape of the log-log plots with varying q is indicative of multifractality (Fig. 3).

Appendix B : Monofractal noise corrupted with sinusoidal trend (y)

The monofractal data with scaling exponent $(\alpha = 0.9)$ was generated using the algorithm of Makse et al., 1996. This data was recently used to study the effect of sinusoidal trends on the detrended fluctuation analysis. The data is publicly available at
http://www.physionet.org/physiobank/database/synthetic/tns/.
This data was corrupted with sinusoidal trend given by $t_i(n) = A_i \sin(2\pi n/T_i), \ n = 1 \ldots N$. to obtain y.

Data 1 : Synthetic

$$y(n) = s(n) + A_1 \sin(2\pi n/T_1), \ n = 1 \dots N, \ A_1 = 2, T_1 = 2^7$$

shown in Fig.1.

Data 2 : Synthetic

$$y(n) = s(n) + A_1 \sin(2\pi n/T_1) + A_2 \sin(2\pi n/T_2) + A_3 \sin(2\pi n/T_3)$$

with $A_1 = 6$, $A_2 = 3$, $A_3 = 2$, $T_1 = 2^6$, $T_2 = 2^4$, $T_3 = 2^2$, shown in Fig. 2. It should be noted that $T_1 = 16T_3$ and $T_2 = 4T_3$.

Appendix C :Multifractal noise corrupted with sinusoidal trend (y)

Data 3 : Real world data The multifractal data (s) considered is that of internet log traffic [Levy et al., 1996].

$$y(n) = s(n) + A_1 \sin(2\pi n/T_1) + A_2 \sin(2\pi n/T_2), n = 1 \dots N, N = 2^{15}.$$

with $A_1 = 6000$, $A_2 = 3000$, $A_3 = 2$, $T_1 = 2^6$, $T_2 = 2^4$.

It should be noted that $T_1 = 4T_2$ as shown in Fig.3.